Cypriot Journal of Educational Sciences

Volume 16, Issue 2, (2021) 827-846

www.cjes.eu

GeoGebra and students' learning achievement in trigonometric functions graphs representations and interpretations

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Suggested Citation:

Mosese, N, & Ogbonnaya, U. I. (2021). GeoGebra and students' learning achievement in trigonometric functions graphs representations and interpretations. *Cypriot Journal of Educational Science*. *16*(2), 827-846. https://doi.org/10.18844/cjes.v16i2.5685

Received from December 12, 2020; revised from January 15, 2021; accepted from April 10, 2021.

Selection and peer review under responsibility of Prof. Dr. Huseyin Uzunboylu, Higher Education Planning, Supervision, Accreditation and Coordination Board, Cyprus.

Abstract

Making connections between the representations of trigonometric functions and an interpretation of graphs of the functions are major challenges to many students. This study explores the effectiveness of the GeoGebra on grade 12 students' success in making connections between the representations of trigonometric functions and the interpretation of graphs. A non-equivalent control-group pre-test post-test quasi-experimental design was used. The sample of the study consisted of sixty-one grade 12 students from two schools. The results showed that there was a statistically significant difference between the mean achievements of the experimental group and the control group on making connections between representations of trigonometric functions, and on analyses and interpretations of representations of trigonometric functions, in favour of the experimental group. This study extends the findings of previous studies on the effectiveness of dynamic mathematics software on students' learning of representations and interpretation of graphs of trigonometric functions.

Keywords: GeoGebra, functions graphs, Trigonometric functions

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1. Introduction

Trigonometric functions are some of the sections in the mathematics curriculum where students experience considerable difficulties in learning (Adamek et al., 2005; Demir, 2012; Ogbonnaya, 2011). The poor performance of students in mathematics (especially in trigonometry functions) is a significant problem and is of high concern nationally (Department of Basic Education, 2016; Sinyosi, 2015). This concern is because trigonometry is one of the cardinal subjects in the mathematics high school curriculum requiring the integration of algebraic, geometric and graphical reasoning (Department of Basic Education, 2011; Stols, 2011).

Some of the problems encountered by students in trigonometry include difficulties in making connections between the representations of the concepts (Elia & Spyrou, 2006). Brown (2005) asserts that students had incomplete or fragmented ways to view trigonometric functions and difficulty in interpreting graphs of the functions.

Although many students fail mathematics, especially trigonometry, research on the teaching and learning of trigonometry is scarce. Davis (2005) observed that regardless of the importance of trigonometry functions in the mathematics curriculum and the difficulties that students experience with them, little attention has been given to trigonometry and the various ways it is taught in the classroom. Ross et al. (2011a) add that research on the teaching and learning of trigonometry, with or without technological aids, lags behind research conducted in other domains of mathematics education.

This study is significant for two main reasons. Firstly, the study was inspired by the need to find an alternative approach to teaching mathematics to improve students' performance. There is pressure on the education sector to find ways of improving learning outcomes in scarce skills such as mathematics, science and technology. Thus, this study was to evaluate the possible influence of GeoGebra in the teaching and learning of trigonometric functions. Secondly, only a few studies have dealt with evaluating the effectiveness of using Information and Communication Technology (ICT) in the teaching and learning of trigonometric functions, although it has often been reported as a difficult topic for students (Brown, 2005; Demir, 2012; Weber, 2005). Besides, the researchers could not find any study in South Africa that emphasizes connections and interpretations of trigonometric functions. Since research on the use of ICT in the teaching of trigonometry in South Africa in the classroom is sparse and quite limited, this study addresses that gap. Currently, trigonometry functions are taught using chalk and talk. Many research studies have acknowledged that students encounter difficulties in learning trigonometric functions (Bornstein, 2020; Demir, 2012; Maknun et al., 2020). In recent times the impact of the use of technology in teaching some difficult mathematics concepts on students' learning has attracted the interest of many researchers, however, how learning technology, and GeoGebra in particular, impacts on students learning of trigonometric functions graphs have not been well explored. Trigonometric functions graphs representations and interpretations are very significant aspects of trigonometric functions.

The purpose of this study was to determine the possible effects of using GeoGebra on students' ability to make connections between multiple representations of trigonometric functions and their interpretation of graphs of trigonometric functions. Thus, the study investigated whether this learning method using GeoGebra surpassed the traditional method. The following questions were posed: Does the use of GeoGebra in the teaching and learning of trigonometric functions affect students' achievement in 1) Making a connection between representations of the trigonometric functions 2) Analysis and interpretation of the graphs of trigonometric functions?

Teaching and learning with the use of technology have many advantages such as providing transformation and new learning possibilities for students; enhancing student engagement and encouraging mathematical investigations and conceptual development (Wilson & Lowry, 2000). In the teaching and learning of mathematics, especially trigonometric functions, students need to be able to imagine, construct and understand the construction of shapes in order to connect them with related facts. Therefore, a computer graphing software solution will assist students in imagining and making observations (Dogan & Icel, 2010).

GeoGebra is an open-source (free) Dynamic Geometry software solution that provides a visual learning environment for students. It integrates possibilities of both dynamic geometry and computer algebra in one program for mathematics teaching. In addition, GeoGebra accepts geometric, algebraic and calculus commands, and it is also able to link multiple representations (Dikovic, 2009; Hohenwarter & Jones, 2007).

2. Literature Review

Jonassen and Rohrer-Murphy (1999) suggest that students will learn trigonometric functions better and more conceptually if they can inter-relate numerical and symbolic representations with their graphical outputs. In line with the suggestion, Brown (2005) developed a model on students' understanding of sine and cosine functions of angles measured in degrees. From her model, the trigonometric functions were first taught using the triangle context, followed by the coordinate system, moving onto the unit circle context and the graphical representations. It should be stated that the steps of introducing the trigonometric function concepts in her model, were non-sequential or they were non-linear. Brown (2005) asserted that students who developed the most robust understanding were able to work with the sine and cosine in a way that connected the three contexts of trigonometry functions. ICT may provide students with opportunities to explore different representations of mathematical ideas and support them in making connections/relationships among different representations of mathematics (Kaput, 1986; NRC, 2000).

Internationally, some studies which incorporated ICT into the teaching of trigonometry, even though sparse, have demonstrated positive effects on student achievement (Moore, 2009; Ross et al., 2011b; Zengin et al., 2012). Although the studies show positive effects of the use of ICT on students' learning and performance, the possible effects of ICT on students' ability to make connections between different representations and contexts of trigonometric functions are limited, even though this is basic and or fundamental to the learning of trigonometric functions (Demir, 2012). Studies on the interpretation and analysis of trigonometric functions were also found to be limited. In addition, the researchers could not find any study that puts focus on the tan functions. This study focuses on connections between the interpretation and transformation of trigonometric functions. Moreover, the tan function was also included.

Though there are some positive indications of the effect of integration of ICT with mathematics teaching and learning in some research studies, some other studies do not show any positive effect of ICT integration with mathematics teaching on students learning outcomes (Smith & Hardman, 2014). The inconsistent findings point to the need for more studies on ICT integration with mathematics teaching and caution in the implementation of ICTs into schools as a potential panacea for mathematical failure.

Using technology in lessons does not automatically lead to better results in terms of students' learning and understanding. Of crucial importance is how technological tools are used in lessons, the kind of support students receive, and interactions between the tools and students. In this regard, Thompson (2002) mentioned the use of technological tools as educational objects which promote reflective mathematics discourse for knowledge construction, but that an object is not instructive on its own.

One has to then consider, the sequence for integrating ICT in the class and the theoretical framework. The optimal sequence for integrating ICT with mathematics instruction has not yet been determined (Ross et al., 2011a). Lesser and Tchoshanov (2005) presented evidence that students need to be taught abstract, visual and concrete representations to develop function sense (the ability to integrate and flexibly apply multiple representations of functions). They found that the working sequence for introducing representations in trigonometry was to present the abstract first; the visual and concrete became meaningful only after the abstract had been learned. This form of a sequence of teaching trigonometry in technological environments was substantiated by Ross et al. (2011a), who concluded that better learning is promoted when ICT is used after the teacher explains the content. They found that, in the case of transformations of the trigonometric functions, using a dynamic software package, after teaching the whole class the core concepts, was more effective than beginning the learning unit with the software.

In this digital era, students are constantly exposed to and actively involved in the use of ICT in their everyday lives (Lopez-Morteo & Lopez, 2007). Research shows that many students exposed to ICT advocate for its integration into mathematics teaching and learning (De Villiers, 2004) and seem to be more motivated to learn (Shelly et al., 2008; Tall, 2000), and have higher learning achievements (Mushipe & Ogbonnaya, 2019).

Dogan and Icel (2010) evaluated the success of students' learning using GeoGebra and found that the software encouraged higher-order thinking skills. The software was also observed as having a positive effect on motivating students toward learning and retaining knowledge for a longer period.

3. Theoretical framework

This study is underpinned by the constructivist theory of learning which draws on the work of Piaget (1977). Constructivist theory of learning posits that knowledge is constructed in an individual's mind by active participation in certain experiences(Roblyer & Doering, 2006). Constructivists believe that people learn best when they gain knowledge through exploration and active participation. Constructivism sees learning as an active process in which students, through a meaning-making search, make sense of their world (Adams, 2006). Similarly, Christie (cited in (Amineh & Asl, 2015)) states that constructivism is a learning theory in which knowledge is viewed as a human construction; learning is both an active process and a personal representation of the world. Christie argues for a problem-solving approach to the teaching and learning process, stating that knowledge is constructed from the experience of problem solving.

From the constructivist perspective, "learning is understood to be a self-regulated process of resolving inner conflicts that become apparent through concrete experience, discussion, and reflection" (Brooks & Brooks, cited in (Gilakjani et al., 2013, p.50). Hence, constructivist learning theory emphasises student-centred instructional methods (Slavin, 2006). Though there are varied theoretical perspectives of constructivism leading to many definitions of constructivism, the varied theoretical perspectives all have the common characteristics that knowledge is constructed and not passively absorbed, people create knowledge by relating new information to previous knowledge; knowledge is created through personal experiences, learning involves active cognitive activity and cognitive growth is stimulated meaningfully through challenging problems for the learner to solve (Amarin & Ghishan, 2013; Richardson, 2003).

The integration of technology (e.g. GeoGebra) with the teaching and learning of mathematics aligns with the constructivist theory of learning. The constructivism theory states that learning is an active process; people learn through exploration and active participation. Technology integration with teaching and learning mathematics enables students' active engagement with the learning as they strive to make sense of the mathematics concepts using technology. Technology, such as GeoGebra, stimulates students to make conjectures and explore the outcome of their conjectures. The explorations with the technological tool lead to reflection and knowledge construction in line with the constructivist perspective.

Also, the integration of technology into the classroom environment creates a student-centred learning environment in accord with the constructivist view of learning. This, according to Gilakjani et al. (2013), "is due in part to the replacement of the traditional seat-work with the use of computers as learning tools. Instead of the static teacher-centred environment where the students act as receivers of information from the teacher, the classroom becomes an active setting full of meaningful activity where the student is made responsible for his or her learning".

4. Method

This research study was conducted using a quantitative approach and followed a non-equivalent pre-test post-test control group quasi-experimental design (Creswell, 2014). The sample of the study consisted of sixty-one students from two separate full-time schools. One school was the control and the other was the experimental group. The schools were in the North West Province, South Africa. These were convenient schools, selected based on their accessibility to the researchers and computer availability at the experimental group school. The experimental group had twenty-seven students whilst the control group had thirty-four students. Both schools were comparable in terms of the socio-economic status of their students, their students' performances in the grade 12 school certificate (matric) examinations in the past years, and their teachers' qualifications and teaching experiences. Also, both schools used the same curriculum compliant to the Matric specifications and used the Department of Basic Education recommended textbooks in teaching and learning mathematics. To avoid disturbing the regular daily running of the school classes, the research was conducted during the normal school periods using the content as prescribed in the curriculum.

4.1 Instrument

The instrument for data collection was a trigonometric function achievement test. The pre-test was administered before the intervention and the post-test was administered after the implementation of the intervention. The achievement test's four questions were open-ended. Two questions (with sub-questions) were on making connections between representations of trigonometric functions, while two questions were on analysis and interpretation of trigonometric functions graphs (see appendix).

4.1.1 Development of the test

According to La Marca (2001), to make valid and reliable decisions on students' achievements, a study should use assessments that are aligned with the curriculum standards. This means that there should be a high degree of agreement between the test tasks and subject matter content as identified through government educational standards. The test questions were thus constructed by using the specification and clarification of the content of the trigonometric functions guided by the Curriculum and Assessment Policy Statement (Grade 10-12) and the mathematics examination guidelines (Department of Basic Education, 2015) which stated that the purpose of the clarification of the topics was to give guidance to the teacher in terms of depth of content necessary for examination purposes.

It should be noted here that some of the questions for the post-test were slightly modified to avoid memorisation of the solutions from the pre-test and to see the improvement of students in the performance of the concepts (McKnight et al., 2000). For example, in the pre-test students were required to draw $h(x) = 3sinx - 2$. In the post-test, the question was to draw $h(x) = 3sinx + 2$.

4.1.2 Validity of the test

Content validity; the degree to which the content of an instrument covers the extent and depth of the topic it is supposed to cover (Creswell, 2008), was determined for the test. Content validity is most often measured by relying on the knowledge of people who are familiar with the concepts being measured. Firstly, in line with the suggestion of McKnight et al. (2000), one of the authors used her professional experience and judgement as a mathematics educator with a lot of teaching experience at the grade 12 level to assess the validity of the test. Secondly, the researchers involved five (5) other subject matter experts to assess the content validity of the tests. The experts consisted of two high school mathematics educators, two high school mathematics head-of-departments (HODs) and one mathematics subject advisor. The experts were asked to determine whether the content reflected the content domain and how well each question measured the concepts in question. Their responses were then statistically analysed (Creswell, 2008; Lawshe, 1975). Lawshe's method of measuring content validity relies on expert responses to each item as being essential or not essential to the performance of the concept (Lawshe, 1975). Accordingly, if more than half the panellists determine an item as essential, then that item has some content validity. When larger numbers of panel members agree that a particular item is essential then the item has greater levels of content validity. Using these assumptions Lawshe (1975) developed the content validity ratio, CVR:

$CVR = (n_e - N/2)/(N/2)$, Where $CVR =$ content validity ratio, $n_e =$ number of panelists indicating that the item was essential, $N =$ total number of panellists. This formula yields values which range from +1 to -1; positive values ≥ 0.5 indicate that at least half the panellists rated the item as essential. The content validity index (CVI), which is the mean CVR across items was used as an indicator of overall test content validity (Lawshe, 1975). From the experts' responses, the content validity was calculated. The minimum CVR value obtained was 0.8 and the CVI was 0.99. Hence, the instruments were considered to be highly relevant and valid.

4.1.3 Reliability of the test

According to Gay and Airasian (2003), for a cognitive test in which the questions are not scored dichotomously, the reliability can be calculated by using the Spearman-Brown formula, R = 2r/(1+r), where r is the correlation coefficient between split-half test results, or between test and the re-test results, or between two equivalent randomly assigned groups. In this study, two randomly assigned groups were used. The correlation coefficient must be significant at 95% or a higher confidence interval (Cohen et al., 2007). The reliability coefficient of the test using the Spearman-Brown formula was 0.81. The results obtained imply that the instrument was very reliable.

4.2 Teaching in the control group

In the control group, the trigonometry triangle context method was used to introduce the subject.

Lesson 1

Firstly, students were taught to define the basic trigonometry functions of angles (sine, cosine and tangent) and reciprocals (cosec, sec and cot) from a right-angled triangle.

Thereafter, angles and the change of angles made by rotation of a vector arm in a clockwise and or anti-clockwise direction were presented and explained. A mnemonic, CAST- (Cos, All, Sin, Tan), was used to show the signs of the basic trigonometry functions in the different quadrants. Just before class activities were assigned, a few examples were given by the teacher. When working with a Cartesian plane, the educator emphasised that the students should first join the terminal end of the vector arm to the x-axis to make a right-angled triangle. Students were allowed to work in groups.

Lesson 2

Special angles diagrams were drawn and explained. One example of a task using special angles was worked on by the educator as a demonstration. Square identities were derived and explained. Examples that involved simplification of trigonometry expressions and the solving of basic equations were shown to students. The students were then given exercises to work on. Solutions were given on the board. More complex tasks were given to students as homework.

Lesson 3

At the beginning of the lesson, corrections from the previous day's homework were reviewed, written and explained. Graphical representations of the basic trigonometry functions were presented. The properties of the functions were also described. Students were then taught how to plot the trigonometry functions whilst using the table method. Class activities were given and the educator assisted the students in the plotting of various graphs. Homework that required the plotting of various graphs and the analysis thereof, was given.

Lesson 4

Corrections of homework from the previous day were given and explained using translation. Tasks on the plotting of graphs, translation and derivation of formulae from given graphs were given. Students were encouraged to work in groups. Corrections were given. The lesson ended with students being introduced to reduction formulae and compound angles. Homework on solving and simplifying expressions and equations was given.

Lesson 5

Solutions to the given homework were discussed. Students were shown how to work on tasks that required general solutions, i.e., $\theta = \alpha + k360°; k \in \mathbb{Z}$. Class activities on simplifying expressions, proving identities and determining general and specific solutions of given equations were given and worked on in the classroom.

4.3 Teaching in the experimental group

At the beginning of each lesson, the educator placed an outline of what was going to be covered on the blackboard.

Lesson 1

At the beginning, the white-board, projector and computers were used for the introduction of trigonometry concepts and representations. The educator then introduced GeoGebra and its facets using some applets. This was then followed by the students working on the computers both individually and in pairs. The applet was used to introduce quadrants, angles, basic, trigonometry functions per quadrant, and trigonometry functions on general angles.

Lesson 2

Homework was first corrected in class. After that, an Applet was used to demonstrate how to plot trigonometry functions graphs. Thereafter students were seen trying out a variety of graphs. Students then drew different kinds of graphs. They were given tasks to assist them in developing the required objectives.

Students had to use two applets on their computers:

$y = asin(x + b) + c$

$y = acos(x + b) + c$

From here they were required to determine the characteristics of the graphs such as the amplitude, intercepts, period, range and domain. Thereafter students were seen trying out a variety of graphs among themselves while constantly receiving assistance from the educator.

Homework was then given on the plotting and transformation for the tan function.

Lesson 3

Corrections to the homework on tan functions were done by the educator and students. The students were then given tasks to analyse relationships between different functions such as points of intersection and $f(x) < g(x)$. Figure 1 is an example of the applets which were given to students to work on.

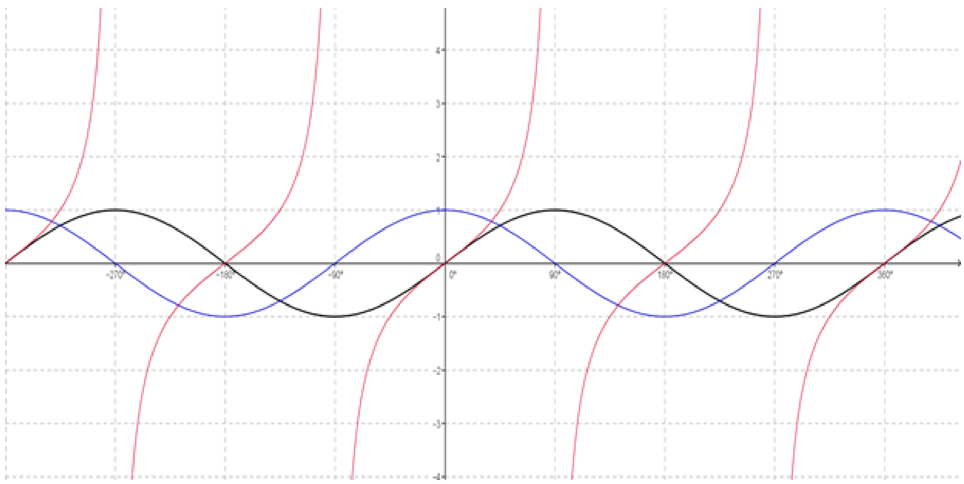

Figure 1. An example used for the analysis of trigonometry functions graphs

They played around with different kinds of graphs, discussed them in class and then worked on their own sets of graphs and analyses.

Lessons 4 and 5

From the computer laboratory, the students moved to their normal classroom, where the teacher used the whiteboard and the projector to discuss and explain the various concepts. Students were still seen to be actively working individually or in groups of four or five.

In these lessons the following topics were dealt with: (i) simplifying expressions, (ii) general solutions and solving equations were dealt with here, and (iii) reduction formulae, negative angles, compound angles were used in all of this.

Self- and peer-assessments were used during the last two lessons.

4.4 Data Analysis

Quantitative data analyses were done using independent-samples t-tests. This was used because the data were found to be normally distributed Moreover, the t-test is the most commonly used test in mathematics education that involves two groups of small sample sizes (McKnight et al., 2000) besides, the t-test is more favourable for a two-grouped sample. Computationally, the statistical software package Statistical Package for the Social Sciences (SPSS) (version 23) was used.

5. Results

The results of this study were presented according to the research questions of evaluating whether any significant difference existed between the pre-test scores and the post-tests of both the control and treatment groups.

5.1 Comparison of the two groups' achievement in the pre-test

Tables 1a and 1b show the $t-test$ comparison of the groups' achievements in the pre-test. The result shows that no statistically significant difference existed between the pre-test scores of the control (M = 13.52; SD. 9.45) and the experimental groups (M = 15.8519; SD. 9.29); t(59) = .96, p >.05. This suggests that the students in the control and experimental groups were comparable in abilities before the treatments were administered.

Table 1a. Groups' mean achievement in the pre-test

	N	Mean	Std. Deviation	Std. Error Mean
Experimental	27	15.8519	9.29724	1.78926
Control	34	13.5294	9.45239	1.62107

Table 1b. A t-test analyses of the students' achievement in the pre-test

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper

Equal variances assumed	.011	.916	.960	59	.341	2.32244	2.41906	-2.51809	7.16297
Equal variances not assumed			.962	56.310	.340	2.32244	2.41440	-2.51359	7.15847

5.2 Comparison of the two groups' achievement in the post-test

The achievement of both groups in the post-test on making connections between representations of trigonometric functions and also on interpretations of trigonometric functions was compared.

5.2.1 Making connections between representations of trigonometric functions

Tables 2a and 2b show the $t-test$ comparison of the groups' achievements on making connections between representations of trigonometric functions. The result shows that there was a statistically significant difference between the achievement of the experimental group (M= 14.41, SD = 2.978) and the control group (M= 7.65, SD = 4.74); t (59) = 6.47, p < .05. The result suggests that GeoGebra does affect students' ability to make connections between representations of trigonometric functions. Specifically, the result suggests that when students are taught trigonometry using GeoGebra their ability to make connections between representations of trigonometric functions will likely be more than when they are taught using the traditional chalk and talk method.

Table 2a. Students' mean achievement in making connections between trigonometric function representations

	N	Mean	Std. Deviation	Std. Error Mean
Experimental	27	14.41	2.978	.573
Control	34	7.65	4.735	.812

Table 2b. A t-test on students' achievement on making connections between trigonometry functions representations

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig 2-tailed	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	1.684	.199	6.467	59	.000	6.760	1.045	4.669	8.852
Equal variances not assumed			6.802	56.324	.000	6.760	.994	4.770	8.751

5.2.2 Analyses and interpretations of trigonometric functions graphs

Tables 3a and 3b show the $t-test$ comparison of the groups' achievements on the interpretation of representations of trigonometric functions. The result shows that there was a statistically significant difference between the achievements of the experimental group (M= 10.89, SD = 4.87) and the control group (M= 7.74, SD = 5.93); t (59) = 2.23, p .05.

The result suggests that GeoGebra does affect students' ability to analyse and interpret graphs of the functions. Specifically, the result suggests that when students are taught trigonometry using GeoGebra their ability to analyse and interpret graphs of the functions will likely be more than when they are taught using the traditional chalk and talk method.

Table 3a. Students' mean achievement on interpretations of trigonometric functions

	N	Mean	Std. Deviation	Std. Error Mean
Experimental	27	10.89	4.87	0.937
Control	34	7.74	5.925	1.016

Table 3b. A t-test on students' interpretations of trigonometric functions

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	2.044	.158	2.230	59	.030	3.154	1.414	.324	5.983
Equal variances not assumed			2.281	58.914	.026	3.154	1.382	.387	5.920

6. Discussion

The GeoGebra software can be used as a facilitation tool in the teaching and learning of mathematics, and more specifically of trigonometric functions, as there was a significant difference in the mean achievement of experimental students on trigonometric functions as compared to the control group. The use of the GeoGebra software not only increased student scores, but it was also observed that the software enabled realisation of a vibrant classroom where cooperative and collaborative activities of learning were evident. This finding is supported by Bakar et al. (2010) along with Zengin et al. (2012). The above findings also corroborate other studies done to determine the effects of a technology-rich environment on students' learning (Dogan & Icel, 2010; Ogbonnaya, 2010). This improvement can be attributed to the social constructivist learning environment in the experimental group which stimulated students to interact, make conjectures and eventually knowledge construction (Vygotsky, 1978).

On connections of trigonometric functions, the experimental group was found to have scored higher than the control group. Here we can deduce that the use of GeoGebra is effective in improving the students' ability to make connections between different representations and contexts of trigonometry functions. Most of the students in this study managed to plot the graphs, which is in contradiction to Demir (2012) who found in his study that most students could not draw the cosine graphs. Similar to the results of Demir (2012), many students in this study could connect a point on the unit circle to a point on a graph, which is in contrast with the findings of Brown (2005).

Concerning interpretation and analysis of trigonometric functions, the experimental group was found to have performed significantly better than the control group. Here we can deduce that the use of GeoGebra was effective in improving the students' ability to interpret and analyse trigonometric functions. During the lesson in the experimental group, the students only needed to type in equations that produced different trigonometry graphs. This gave them time to explore, investigate and interpret the properties of the different graphs. This was unlike situations where students would have to draw graphs manually from point to point and then analyse them. This is confirmed by Clements (2000) who stated that instant feedback from ICT programs encourages students to use conjectures and to keep exploring.

7. Conclusion

The results of this study showed that GeoGebra assisted instruction in the teaching of trigonometry functions and had significant effects on students' achievement. Specifically, the GeoGebra assisted instruction was found to be more effective than the traditional chalk and talk method of teaching on improving students' achievement in making connections between representations of trigonometric functions and also in the interpretation of trigonometric function graphs. GeoGebra helped the students in the experimental group to have a better understanding of representations of trigonometric functions and graphs. The GeoGebra assisted instruction made the students gain more knowledge through exploration and active participation than the traditional teaching method.

The findings of this study suggest several implications for teaching and learning mathematics in general and trigonometric functions in particular. The current study is the first intervention study in South Africa designed to investigate the effect of GeoGebra in the teaching and learning of trigonometry with a focus on connections between representations and interpretation of trigonometric functions. First, we recommend that more studies be conducted on the effect of GeoGebra assisted instruction on students' achievement with a larger sample of students, at different grade levels and on different topics in mathematics. The findings from such large sample studies may be used to corroborate the findings of this study. Second, we recommend that teachers integrate GeoGebra with the teaching of trigonometric functions in particular and mathematics in general. For teachers to integrate GeoGebra into mathematics teaching they may need to be trained on general ICT skills and how to use Geogebra in teaching. Many teachers do not use computer software-assisted instruction because they do not know how to incorporate ICT in teaching. Hence, we also recommend that teacher training institutions should include courses on how to use dynamics software applications to teach mathematics in teacher-training programmes. For the practising teachers, there is a need for them to be given regular in-service training on how to integrate dynamic mathematics software applications in teaching.

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Appendix

Achievement Test

Question 1

Draw, on the same system of axes, the graphs of the following functions:

1.1a $y = f(x) = -3\tan 2x;\ \ x \in [-180°; 180°]$

1.1b $y = g(x) = cos\ (2x); x \in [-180°; 180°]$

1.1c $y = h(x) = 3sinx + 2; x\ \in [-180°; 180°]$
Clearly show all the important points. (12)

Question 2

For each graph, answer the following questions:

a) Write down the amplitude of g. (1)

b) Write down the amplitude of h. (1)

c) Give the period of f. (1)

d) Give the domain of g. (1)

e) Give the range of h. (1)

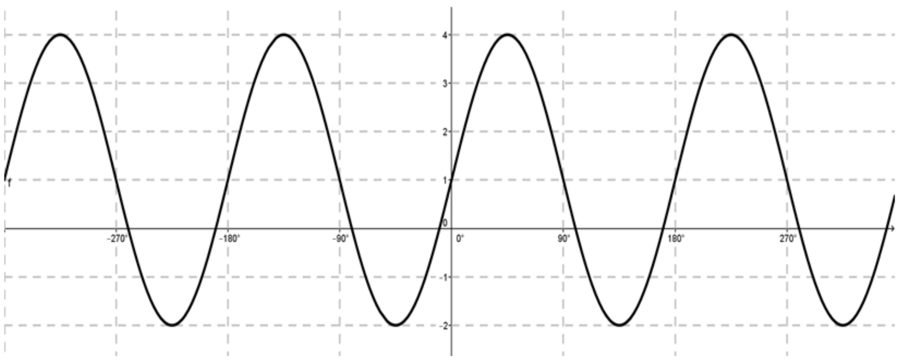
f) Give the maximum value of h. (1)

g) Write down the maximum value of g. (1)

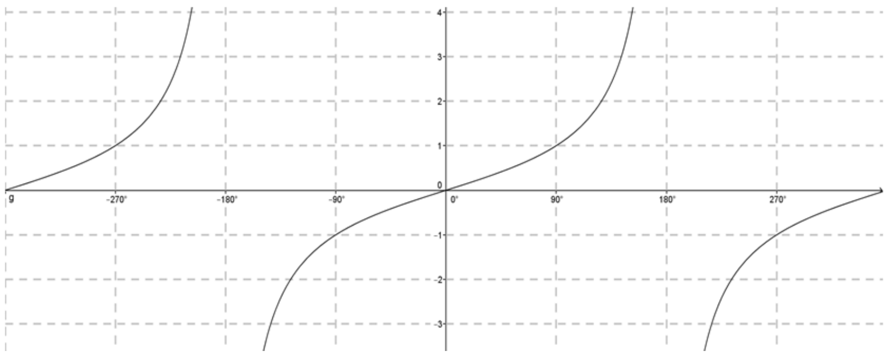
h) Write down the asymptotes of f. (2)

i) Write down the equation of the function of $f(x)$ if it is moved 30° to the left and two units up. (2)

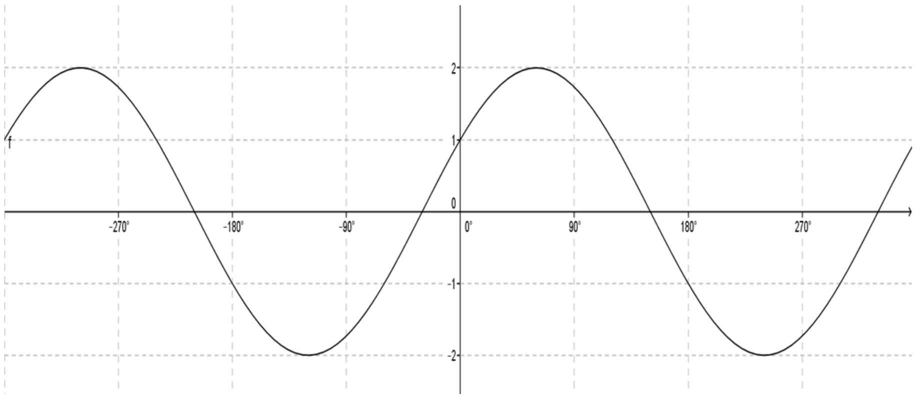
Question 3
Write down the value(s) of a, b, p and q from the graphs below:
a) $y = asin\ bx + q$ (6)



b) $y = a\tan bx$ (3)



c) $y = a\cos(x + p)$ (4)

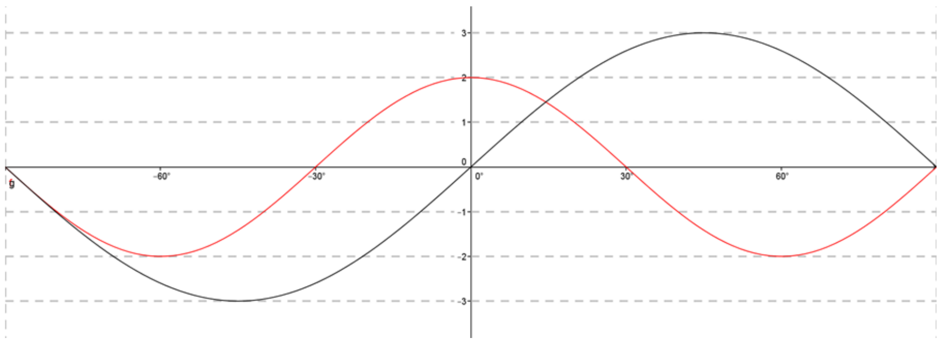


Question 4

The diagram below represents the graphs of $y = f(x) = 2\cos 3x$ and $y = g(x) = 3\sin 2x$ for $x \in [-90°; 90°]$

a) Write down the coordinates of P, the y-intercept of g. (2)

b) Write down the coordinates of the x-intercepts of f and g. (4)



c) On the graph, show where the points of intersection of f and g are. Label the points as S, T and U. (3)

d) From the graph determine the value(s) of (x) for which

i) $g(x) = 3$ (1)

ii) $f(x) = g(x)$ (3)

e) On the graph, shade the regions where $f(x) \geq g(x)$. (3)